Dynamic friction coefficient

1. Introduction

In this laboratory session, we will study how friction affects the uniformly accelerated movement in an inclined plane. The aim of this session is to understand how the acceleration of an object in an inclined plane depends on the action of gravity and on the frictional force. The values of the friction coefficient of the surface can inferred by measuring the velocity of the object at different displacements on the inclined plane.

We will consider the movement of a block of mass M in an inclined plane at an angle θ (see Fig. 1), which is stretched by a second block, of mass m, pending from a rope that goes through a pulley. We will disregard the mass of the pulley and the friction of the rope. The block with mass M will be initially at rest and we will take its position as $d = 0$ m. We will calculate the speed v of the mobile block for different displacements d. The mobile block will experience a uniformly accelerated movement with acceleration a.

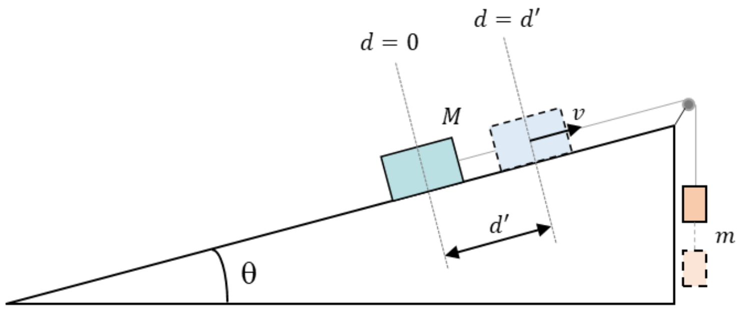

Fig. 1. Displacements d of the mobile mass block M on the inclined plane.

According to the kinematics laws for uniformly accelerated motion, if the initial velocity is equal to zero

$$d = \frac{1}{2}at^2, \quad v = at. \tag{1}$$

Therefore, we obtain

$$v^2 = 2ad \tag{2}$$

In the previous equations, v is the speed of the mobile when it has travelled a distance d along the inclined plane.

The diagram of the forces on the body in the inclined plane, of mass *M*, considers its weight ($P_M = Mg$, with *g* the acceleration of gravity), the force that the inclined plane makes on this mobile body (normal force, F_N), the frictional force that opposes the movement, F_{fr}, and *T*, the tension of the rope from which the mass *m* hangs (see Fig. 2). When the block is in motion, along the direction of the longitudinal axis of the inclined plane two forces will have the same direction and sense: the friction force, which considers the dynamic friction coefficient (μ), and the weight component in that axis. Alternatively, the tension of the rope will be along the same direction but in the opposite direction sense. In the perpendicular direction, the corresponding component of weight and normal force are matched.

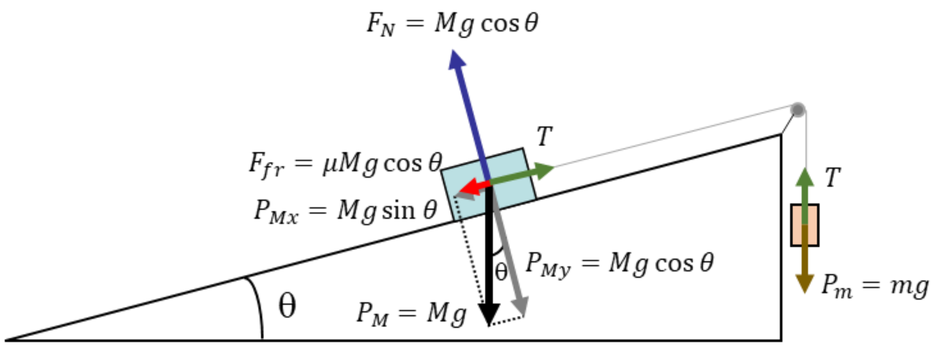

Fig. 2. Diagram of forces of a mobile on the inclined plane.

Mathematically, the relation of forces in the longitudinal axis of the inclined plane, and in the mass *m*, are expressed, respectively:

$$T - \mu Mg\cos\theta - Mg\sin\theta = Ma, \quad mg - T = ma \tag{3}$$

And then, the acceleration in the presence of the friction force is

$$a = \frac{mg - Mg(\sin\theta + \mu\cos\theta)}{M+m} \tag{4}$$

In the absence of friction, the acceleration would be

$$a_0 = \frac{mg - Mg\sin\theta}{M+m} \tag{5}$$

Alternatively, the friction coefficient can be calculated as

$$\mu = \frac{mg - Mg\sin\theta - (M+m)a}{Mg\cos\theta} = \frac{M+m}{Mg\cos\theta}(a_0 - a) \tag{6}$$

Our objective is to measure *v*, for different values of the distance travelled by the mobile block, *d*, and thus determine the experimental acceleration value (with friction) from an adjustment by least squares. From a geometric calculation, the angle of elevation of the lane θ and the friction coefficient μ is calculated.

2. Experimental procedure

First, tilt the rail by placing the box provided under one of the feet of the rail. The angle of inclination depends on the distance L between the feet of the rail (when it has not been tilted) and the elevation h (height of the box) according to the expression $\sin\theta = h / L$. Take a skate and mount the flag (see Fig. 3). Next, mount the pulley and hook the weight support to the skate with the help of the thread.

Fig. 3. Experimental assembly: (left) skate and (right) assembly with the photoelectric cell.

A photoelectric cell will be used to determine the speed of the mobile object, from the measurement of time t that the flag is blocking the light beam of the cell:

$$v = \frac{d_{ef}}{t} \tag{7}$$

where d_{ef} is the effective length of the flag, which you have to measure. A key point in the experiment is the correct speed measurement. **The photocell is placed in a fixed position at the end of the rail**, and the block (the piece of wood) is released in different positions separated by a distance of 10 cm. We will consider as a reference in each position the point at which the flag will begin to cut the light beam of the cell. The measured speed will be interpreted as an average speed corresponding to the width of the flag.

To begin the measurements, hold the block in the different positions, and make sure that the object part from a resting position, $v_0 = 0$, releasing it without initial impulse. Use a mass of 60 g (10 g from the support and another 50 g) hanging from the pulley to induce the movement. **Repeat the experiment** with a mass of **50 g** (10 the support and another 40 g) hanging from the pulley.

For a given distance *d* between the photocell and the resting object, read the time *t*, which measures the photocell, and record the measured values. Take three readings of the time for each output position to reduce the measurement error. Change the starting position of the resting object by 10 cm and repeat the process. Construct a table with the 3 values of *t*, its mean and the velocity *v*.

If we compare the equation $v^2 = 2\,a\,d$ with the equation of a straight line $y = m\,x + n$, we verify that we can identify as independent variable (*x*) the distance traveled *d*, as dependent variable (*y*) at the velocity squared v^2, and as slope (*m*) to product *2a*. The intercept (*n*) would in this case be equal to zero.

Add to the table the values of *x* and *y*, that is *d* and v^2, represent them graphically and calculate the line that best defines their behavior by means of an adjustment by least squares. From the **slope of the adjustment**, obtain the experimental acceleration value and **compare it with the calculation of the acceleration** when there is no friction, a_0 (equation 5). Then, knowing the values of the acceleration of gravity and the angle of inclination of the rail, you must obtain the value of the friction coefficient (from the last term of Eq. 6). Do not forget to weight the values of the masses involved, since they are also necessary.

As a result, you have to provide the requested table, the representation of v^2 as a function of *d* and its adjustment, the comparison between the values of the experimental acceleration and the acceleration without friction, and the value of the friction coefficient with its error, for both cases studied.

3. Additional questions

a) Comment if the experimental acceleration is greater or smaller than a_0.

b) How would the experiment affect the length of the flag being 10 cm instead of the 2.5 cm that you used?