Elastic constant of a spiral spring

1. Introduction

The objective of the practice is to experimentally determine the elastic constant *k* of the spring. For this reason, we will use a setup as that shown in Figure 1 in which a spiral spring is hanging from a holder with a weight hanging at its end.

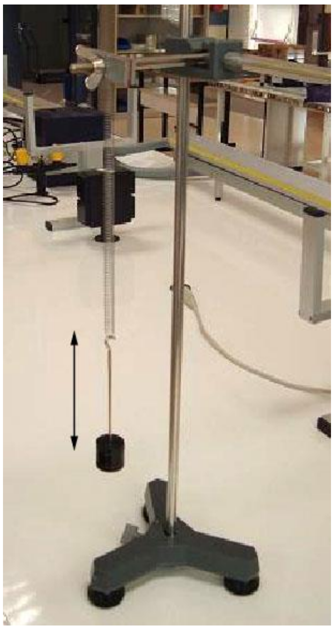

Fig. 1. Experimental setup to study spring oscillations

We will consider the oscillations of the spring with an elastic constant k and mass of the weight *m*. In the case, the frequency of oscillations is

$$\omega = \sqrt{\frac{k}{m}} \qquad (1)$$

The period T and the frequency are related according to

$$T = \frac{2\pi}{\omega} \qquad (2)$$

and it follows that

$$\frac{4\pi^2}{T^2} = \frac{k}{m} \qquad (3)$$

Therefore, the relation between the hanging mass m and the period of the small oscillations T is

$$m = \frac{k}{4\pi^2} T^2 \qquad (4)$$

The objective of the practice is to experimentally determine the constant k of the spring. Using equation 4 we can note that this is the equation of a straight line with $T^2 = x$ and $m = y$

$$y = ax + b \qquad (5)$$

where a is the slope of the straight line and b the intercept with the y axis. Therefore, by measuring the period (T_i) for different masses hanged (m_i) we can generate the data required to plot m vs T^2 which will provide a straight line. The slope of this line can be correlated with the elastic constant k of the spring. The calculation of the best straight line that fits the data will be carried out by least squares, we will calculate a (slope of the straight line) and, hence, we will be able to calculate the value of k.

2. Experimental procedure

Measurement of the period for each mass (***20, 30, 40, 50 and 60 grams***). Each period will be measured as follows: A small oscillation will be applied to the mass perpendicular to the equilibrium position. You must ensure that the oscillations are completely in the vertical direction. Eq. (1) is only valid for small oscillations, therefore, you must apply a *small* force that will give rise to a small amplitude. When the mass oscillates regularly, the time t in which the mass makes 25 complete oscillations will be measured (the measurement will be repeated 3 times).

3. Data processing and presentation of results

The results will be recorded directly in the laboratory notebook. The mean $\bar{t}$, the period T = $\bar{t}$ /25 and the errors of $\varepsilon_{\bar{t}}$, ε_T, ε_{T^2} will be calculated. The results will be presented in a table:

m (g)	t (s)	$\bar{t}$ (s)	$\varepsilon_{\bar{t}}$ (s)	T (s)	ε_T (s)	T^2 (s^2)	ε_{T^2} (s^2)
20	26.2 25.0 26.3	25.83	…	…	…	…	…
…	…	…	…	…	…	…	…

Graphic representation. We will graphically represent the values of $(T_i^{\,2}, m_i)$. We will observe that the points follow a straight line, this line can be approximately plotted. The results should be copied in the laboratory notebook.

Data adjustment. We will do the least squares linear regression with the Excel program of the data $(T_i^{\,2}, m_i)$. We will thus determine the slope *a* and its error ε_a.

Parameters determination. From the result of the previous point, we will determine *k* with its error ε_k. We will give the correctly written result (according to the rules of data processing and calculation of errors), with SI units.

4. Additional questions

Answer the following questions in the lab notebook:

- What is the type of error made when measuring the time (*t*) with the stopwatch? Is it a type of direct or indirect error?
- What type of error is obtained when performing the mean time ($\bar{t}$) for three independent measurements?
- Are errors ε_k and ε_a direct or indirect errors?
- Discuss the advantages of this method for measuring the elastic spring constant
- What other method for determining the elastic constant could be used to measure the elastic constant of a spring?
- What advantages does this method (the method used in this practice) have compared to other existing methods?
- From the calculation of the relative error of *k*, do you think this measurement method is accurate?
- Why do we calculate *k* from the slope and not as an average of the results obtained for each weight?